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Configuration for Propellant Gauging in Satellites

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Accurate propellant gauging is of prime importance in satellite industries. This paper explores the possibility of using a new propellant tank configuration, consisting of a truncated cone centrally mounted within a spherical propellant tank, to measure the amount of liquid propellant present within the tank. The liquid propellant present within the propellant tank orients itself in geometry by virtue of its dominant surface tension force in 0 *g* condition, which minimizes its total surface energy. This study reveals that the amount of liquid propellant present in the tank can thus be estimated by measuring the height of the propellant meniscus within the central cone. It is also observed that, for the proposed configuration, the precision of the estimated propellant fill-fraction increases towards the end of life of the spacecraft.

I. Introduction

ACCURATE determination of the amount of propellant present in a satellite toward its end of life is of prime importance in the determination of exact mission life and to plan a satellite replacement mission well in advance. Also, the annual revenue generated from a typical communication satellite operating at its full capacity is on the order of millions of dollars and, hence, premature removal of spacecraft from their orbits results in heavy losses.

Among the existing techniques, the bookkeeping method and the gas law method are extensively used. The bookkeeping method does not require additional instrumentation but the uncertainties associated with it are quite large; the uncertainty in the determination of mission life is on the order of 10% of total mission life. The propellant gauging system developed by Chobotov and Purohit [1] is better than the bookkeeping method, but an uncertainty analysis using Monte Carlo simulation suggests that the calculated propellant volume using the gas law method overestimates the actual amount of propellant present in the spacecraft [2]. Also, the extent of overprediction seems to increase toward the end of life of the spacecraft.

The major difficulty in the spacecraft propellant gauging stems from the fact that the propellant within the spacecraft tank is in a low-gravity environment. In this paper, an attempt is made to explore the possibility of exploiting the low-gravity condition to assist the propellant gauging. It was found that mounting a cone with its truncated vertex at the exit of the propellant tank, as shown in Fig. 1, orients the propellant in a configuration favorable for measuring its content. The results of the study and the issues addressing the practical feasibility of the technique are discussed.

II. Propellant Configuration Within the Propellant Tank

In a low-gravity environment, the surface tension force of liquid propellant becomes the dominating force determining the final equilibrium configuration. The equilibrium configuration attained by the liquid propellant is the one that minimizes its total surface energy, which is given by [3]

$$E = \sigma(A_f - A_w \cos\theta) \qquad (1)$$

where σ is the interfacial liquid-vapor surface tension parameter and A_f and A_w are, respectively, the areas of liquid-vapor interface and of solid-liquid interface. As a consequence, a column of wetting liquid within a tapered tube tends to migrate toward the converging end of the tube. This behavior of liquid propellant has been made use of by mounting a cone with its truncated vertex at the exit of the propellant tank, as shown in Fig. 1. The propellant is then confined to the conical and annular regions as indicated in Fig. 1. If the propellant residing in the central cone increases, the free surface area of liquid propellant will increase, leading to increase in the surface energy of the liquid propellant. Such is the case with propellant residing in the annular region. Thus, the liquid propellant, both within the central cone and the annular region, stays as close to the cone's truncated vertex, conforming to an axisymmetric configuration and the liquid surface makes the same contact angle θ wherever it meets the tank surface.

III. Propellant Configuration Within the Central Cone

Because the equilibrium free surface attained by the liquid propellant is such that it has a constant mean curvature, the propellant's meniscus FGF' within the central cone (see Fig. 1) is part of a spherical surface.

Using the knowledge of solid geometry, the volume of the propellant contained in the central cone can be derived as

$$V_{\text{cone}} = \frac{\pi r_c^3}{3}\left(\cot\alpha - \frac{2[1-\sin(\alpha+\theta)]}{\cos^3(\alpha+\theta)} + \tan(\alpha+\theta)\right) \qquad (2)$$

where α, θ, and r_c are the semicone angle of the central cone, contact angle of the liquid propellant with the tank material and the radial distance of point F' (which represents the line of contact of liquid propellant meniscus with the cone) from the axis of the cone, respectively. The free surface area of liquid propellant contained in the central cone is given by

$$A_{f1} = 2\pi r_c^2\left(\frac{1-\sin(\alpha+\theta)}{\cos^2(\alpha+\theta)}\right) \qquad (3)$$

and the area of wetted surface formed by the liquid propellant in the central cone is given by

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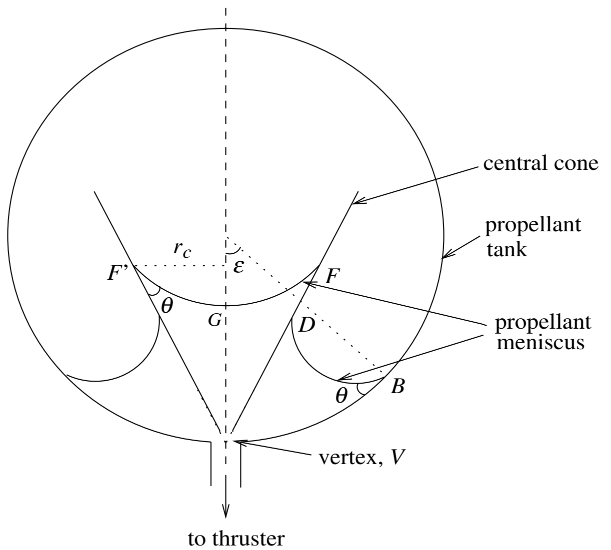

Fig. 1 Liquid propellant configuration in spherical tank with a central cone.

$$A_{w1} = \frac{2\pi r_c^2}{\sin\alpha} \tag{4}$$

IV. Propellant's Free Surface Configuration in the Annular Region

The expected configuration of liquid propellant within the annular region of the propellant tank is axisymmetric in nature (see Fig. 1). Let B be the point representing the line of contact of the liquid propellant meniscus with the spherical tank and let ε be the angle subtended at the center of the tank by point B relative to the axis of the cone (see Fig. 1). Depending on the amount of propellant present in the tank and the relative magnitudes of semicone angle α and contact angle θ, the propellant residing in the annular region can have four different geometries [4]. When $\theta < \alpha$, the curve representing propellant's free surface has a point A where the tangent is parallel to the tank axis. When $\theta < \varepsilon$ (the angular position of point B), the curve representing propellant's free surface has a point C where the tangent is normal to the tank axis (see Fig. 2).

The equilibrium free surface attained by the liquid propellant is such that it has a constant mean curvature H. The inclination ψ of the tangent at an arbitrary point Q lying on the BC part of the meniscus (see Fig. 2) is given by [4]

$$\sin\psi = \frac{Hr(2R_o + r)}{(R_o + r)} \tag{5}$$

where R_o is the radial distance of point C from the tank axis and r is the radial distance of point Q from point C. When the point Q lies on the CA part of the meniscus, the inclination of the tangent is given by (see Fig. 2)

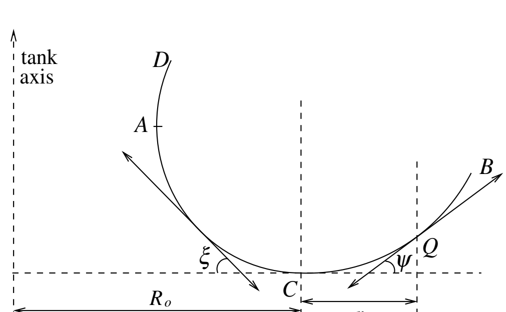

Fig. 2 Free surface of the liquid propellant in the annular region.

$$\sin\xi = -\frac{Hr(2R_o + r)}{(R_o + r)} \tag{6}$$

It should be noted that r is positive when point Q lies on the BC part of the meniscus and negative when it lies on the CA part of the meniscus. The value of mean curvature required in Eqs. (5) and (6) is given by

$$H = \left(\frac{R_b}{R_b^2 - R_o^2}\right)\sin\varepsilon \tag{7}$$

where R_b is the radial distance of point B from the tank axis. The AD part of the meniscus is obtained by taking the mirror image of the relevant part of the curve CA. (For complete derivation and methodology used for obtaining the propellant meniscus in the annular region, see [4].) The curve representing the meniscus, for various positions of point B (parameterized by angle ε), is obtained by numerically solving the differential equation

$$\frac{dy}{dr} = \tan\psi \tag{8}$$

While solving for the BC part of the curve, ψ in Eq. (8) is indicated to ψ given by Eq. (5). However, for solving the AC part of the curve $\psi = \pi - \xi$, where ξ is given by Eq. (6). The correct value of R_o required for the calculation is determined by trial and error by invoking the boundary condition that point D lies on the cone and the meniscus makes an angle equal to the contact angle θ with the cone. The meniscus free surface area A_{f2} and propellant volume V_{annulus} within the annular region are determined numerically using Pappus's centroid theorem [5]. The area of surface wetted by the liquid propellant in the annular region is given by

$$A_{w2} = \frac{\pi R_D^2}{\sin\alpha} + 2\pi a^2(1 - \cos\varepsilon) \tag{9}$$

where R_D is the radial distance of point D from the tank axis and a is the tank radius.

V. Determination of the Equilibrium Propellant Configuration

If V_t is the total volume of propellant present in the tank, then the aim of determining the equilibrium configuration is accomplished simply by finding the two volumes V_{cone} (propellant volumes residing in the central cone) and V_{annulus} (propellant volumes residing in the annular region) such that their sum $V_{\text{cone}} + V_{\text{annulus}} = V_t$, and the total surface energy E [Eq. (1)] corresponding to the propellant volume V_{cone} in the cone and the propellant volume V_{annulus} in the annular region, is minimized. The equilibrium configuration is determined in terms of the parameter ε. It should be noted that a given value of ε corresponds to a unique value of V_{annulus} and because $V_{\text{cone}} = V_t - V_{\text{annulus}}$, the given value of ε also corresponds to a unique value of V_{cone}. Note that once the energy minimizing angle $\varepsilon_{\min}$ (value of ε corresponding to minimum energy E) is determined, the complete equilibrium geometry of the liquid propellant is determinable.

VI. Results and Discussions

Figure 3 shows the equilibrium configurations (determinable through the angle $\varepsilon_{\min}$) for various propellant fill-fractions V_f (ranging from 0 to 0.20). In Fig. 3, the semicone angle α and contact angle θ are 28 and 0 deg, respectively. It may be noted that all liquid propellant used in practice are wetting in nature and have contact angle $\theta = 0$ deg with clean metallic surfaces [6]. The energy E is presented in Fig. 3 in dimensionless form $E/\sigma A_s$ where $A_s = 4\pi a^2$ is the surface area of the spherical tank.

From the knowledge of fluid statics, it is necessary that equilibrium configuration of liquid propellant, as in Fig. 3, corresponds to the configuration having the same mean curvatures for the propellant's free surface within the cone and in the annular

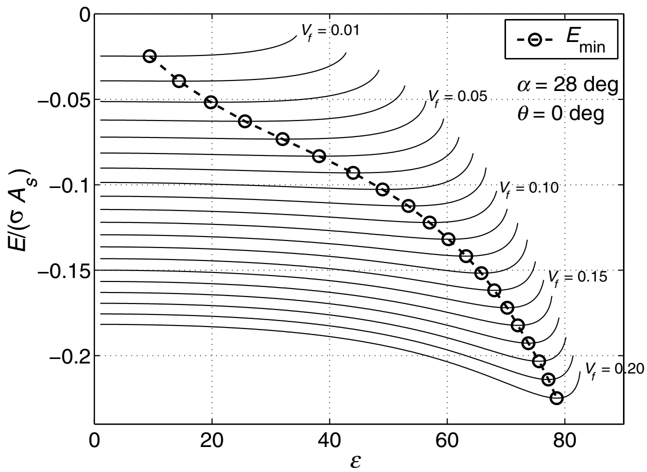

Fig. 3 Variation of ε_{min} with propellant fill-fractions.

region. Figure 4 shows the mean curvature of propellant's free surface within the central cone and the annular region corresponding to the equilibrium configuration obtained in Fig. 3 for different propellant fill-fractions. The two curvatures have been nondimensionalized by dividing them with the curvature of the spherical tank. It is observed that the two curvatures match well for equilibrium configuration corresponding to various propellant fill-fractions, serving as a check of the correctness of the results obtained.

Once the values of ε_{min} for different propellant fill-fractions V_f are known, the volume fractions confined in the annular region and in the central cone under equilibrium can also be estimated. This is shown in Fig. 5. In Fig. 5, $V_{af} = V_{annulus}/(4\pi a^3/3)$ is the volume fraction confined in the annular region and $V_{cf} = V_{cone}/(4\pi a^3/3)$ is the volume fraction confined in the central cone. It may be noted that when the total propellant fill-fraction is relatively high, most of the propellant resides within the annular region of the propellant tank and that residing in the cone is low. However, as the volume fill-fraction decreases, the propellant content within the cone begins to rise. Finally, when the total propellant fill-fraction falls below 10%, the propellant confined within the cone becomes greater than that present in the annular region. This condition persists until all the propellant is exhausted. Further, it can be noted that when the total propellant fill-fraction is 8%, the propellant fraction confined in the central cone attains the maximum value of 5%. It can also be noted that for lower values of propellant fill-fractions, most of the propellant is confined within the central cone and little in the annular region. This qualitative behavior of liquid propellant configuration within the propellant tank with the variation in propellant fill-fraction can all be attributed to the geometry of the propellant tank.

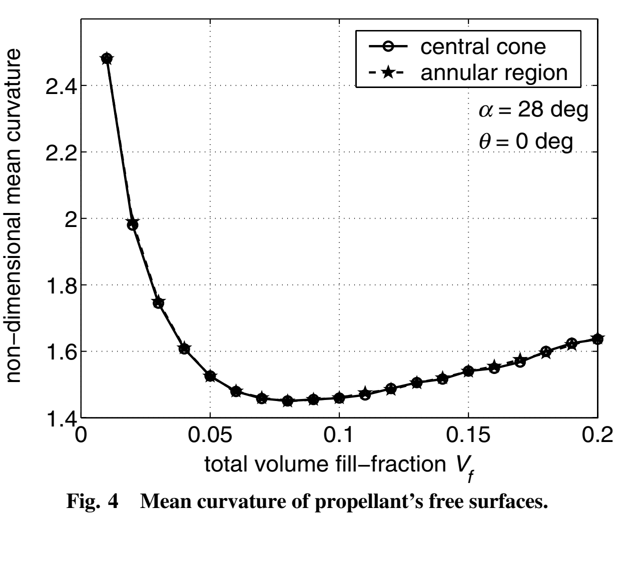

Fig. 4 Mean curvature of propellant's free surfaces.

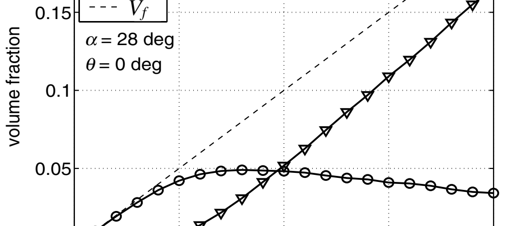

Fig. 5 Variation of V_{cf} and V_{af} with V_f.

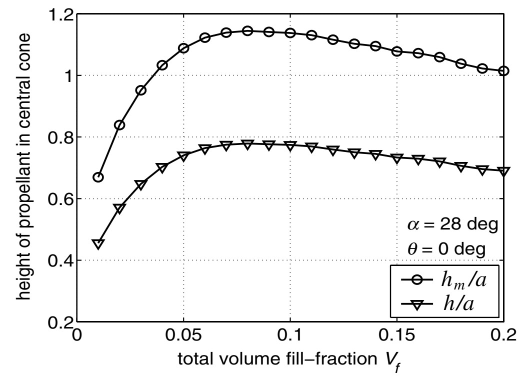

Fig. 6 Height of the propellant meniscus in the central cone.

Figure 6 shows the height of propellant meniscus in the central cone for different propellant fill-fractions. In Fig. 6, h is height VG (see Fig. 1), as given by

$$h = \frac{r_c}{\tan\alpha} - \frac{r_c}{\cos(\alpha+\theta)}[1 - \sin(\alpha+\theta)] \tag{10}$$

and height $h_m = r_c/\tan\alpha$ is the vertical distance of point F above cone vertex V (see Fig. 1). It is seen that, initially, with the decrease in total propellant fill-fraction the height h monotonically increases; when the total propellant fill-fraction is about 8%, the height h attains its maximum value. Later, as the propellant is exhausted from the propellant tank, the height h monotonically decreases. Thus, the variation of height h with propellant fill-fraction is consistent with the observations made earlier with reference to Fig. 5.

The height of the meniscus therefore becomes a good measure of the residual propellant. To check the sensitivity of the proposed configuration to the inaccuracy in measured height h, the slope $\Delta V_f/\Delta h$ is numerically calculated using the data of Fig. 6 (many more data points were used for this calculation than what are shown in Fig. 6). Figure 7 shows the variation of sensitivity $(100 \times \Delta V_f/\Delta h)$ of the proposed configuration with the propellant fill-fraction ($V_f < 0.06$, $\alpha = 28$ deg). It may be noted that the sensitivity of the proposed configuration is pretty high for propellant fill-faction $V_f > 0.05$ but then decreases with the decrease in propellant fill-fraction, ensuring higher accuracy toward the end of life of the spacecraft. The small deviations of the sensitivity value for some propellant fill-fractions seen in Fig. 7 are due to the numerical error in computation of $\Delta V_f/\Delta h$.

The important aspect to be noted here is that, unlike the gas law method [2] or bookkeeping method, in which the accuracy of the estimated propellant fill-fraction decreases toward the end of life of the spacecraft, for the proposed new configuration the accuracy increases. The primary reason for this improvement lies in the fact that the proposed configuration assists to measure the propellant amount directly, whereas the gas law and the bookkeeping methods measure the complement of the residual volume. The gas law method primarily measures the ullage volume, whereas the bookkeeping

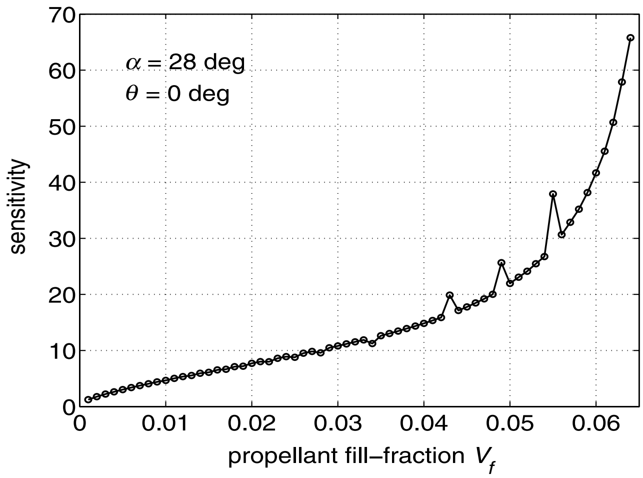

Fig. 7 Effect of uncertainty in measured height *h* on propellant fill-fraction.

method keeps account of propellant consumption history from which the amount of residual propellant is estimated.

VII. Conclusions

A configuration consisting of a truncated cone centrally mounted within a spherical propellant tank, to measure the amount of liquid propellant present within a clean tank, is explored. The major conclusions that can be drawn from the study are

1) The modeling and computational technique adopted here are generic and can be used for many similar configurations.

2) The entire propellant geometry within the propellant tank is the function of the angle ε alone.

3) When the volume fill-fraction is relatively high, most of the propellant resides within the annular region of the propellant tank. The reverse is true for low propellant fill-fractions.

4) The accuracy of the proposed configuration improves toward the end of life of the spacecraft.

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